

THE DIVISION MACHINE

<https://scratch.mit.edu/projects/515829982>



One way to look at DIVISION is: **a way to find how many times** you can subtract a number from a bigger number until the big number has become too small to subtract from it one more time. See it in action in the **Division Machine**.

1. Read. Click the Start Button.

2. Input a Dividend and a Divisor.

3. The *Subtraction ACTION* starts.

Example. $239 \div 27 = 8$ remainder 23

There are **three numbers** in every division calculation. Each number has a special name:

- The starting big number is the **DIVIDEND**.
- The number you are subtracting is the **DIVISOR**.
- The number of times you can subtract is the **QUOTIENT**.

Sometimes there is another number. It's what's *left of the dividend*, when it has become too small to subtract the divisor one more time.

- That number is called the **REMAINDER**.

What is the Division Machine?

This is a teaching resource. You can ignore the SCRATCH code that runs it, unless you are curious! The **Division Machine** shows **Division as continuous Subtraction**. It uses ordinary language to simplify the maths terms. The BIG NUMBER (is the Dividend). The TAKE-AWAY number (is the Divisor). The subtr-ACTION list (is the action of continuous subtraction). The LENGTH of the list returns the number of times you have subtracted the divisor from the dividend. It returns the QUOTIENT. The REMAINDER is what's left of the dividend when it is smaller than the divisor (in other words, it's too small to make another subtr-action).

There is a Task at end:

Explain what you get when you **multiply** the **Divisor** by the **Quotient** and then add the **Remainder**.

Try short division.

$$60 \div 12 = ?$$

Try long division.

$$274 \div 28 = ?$$

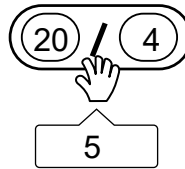
You get the dividend.



Division is presented in ways that look different. They mean the same thing.

$$20 \div 4$$

A short line with a dot above and another dot below with the **dividend** first and the **divisor** second. The **quotient** (answer) here is 5.



The Operator block in SCRATCH has a forward slash that separates the **dividend** from the **divisor**. This example reports the **quotient** value of 5.

Written division is shown like this when dividing by a single digit **divisor**.

$$4 \overline{)20}$$

The forward slash is often used on a Calculator.



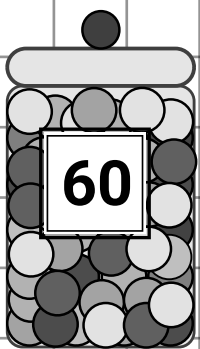
$$\begin{array}{r} 18 \\ 17 \overline{)318} \\ \underline{-17} \\ 148 \\ \underline{-136} \\ 12 \end{array}$$

This is **long division**. It requires subtraction below the **dividend**. It looks like this. You have to make a rough work list on the side and make estimates. See next page.

Division is often presented as a **top-heavy** fraction, because it is related to fractions. The terms used are **numerator** on top and **denominator** below.

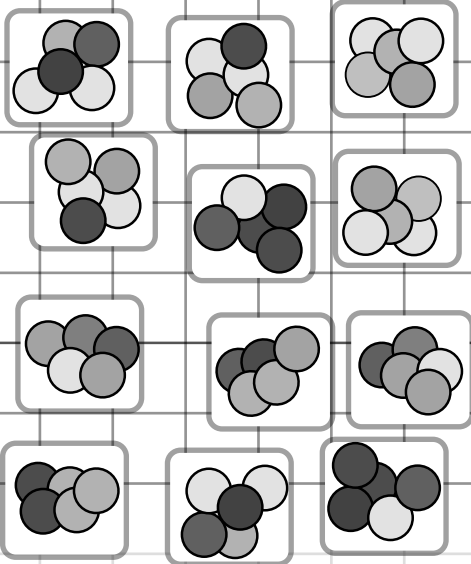
$$\frac{20}{4} \text{ OR } \frac{318}{17}$$

DIVISION is related to MULTIPLICATION



This full sweet jar says that it contains 60 sweets. If the sweets are shared equally among 12 children, each will receive 5 and the jar will be empty. That's because 12 times 5 equals 60. This tells us another interesting fact about division, it is the reverse of multiplication.

DIVISION is Equal Sharing



If you were organising a party for 12 children and each child got a 'goodie' bag. Among the items in each bag was an equal share of all the sweets from this jar.

Each child would get 5 sweets



DIVISION is Repeated Subtraction

If it was done differently and each child at the party was to take 5 sweets from the jar. There would be enough sweets for the last child to get 5 like the rest.

Each child would still get 5 sweets

Knowing what division means is one thing. Knowing how you **do** division is another thing. If the numbers are low (less than 100) you can do division **in your head** if you know the **division facts**. If you know the **MULTIPLICATION FACTS** up to 10 x 10, you can quite easily turn them into **DIVISION FACTS** because they're just the **reverse**.

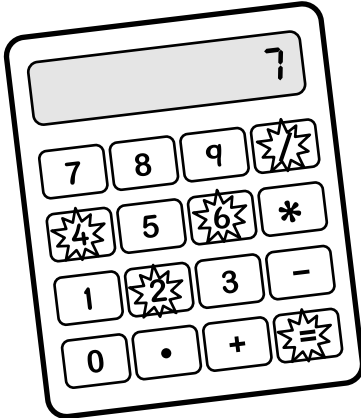
$$12 \times 5 = 60 \quad 60 \div 5 = 12$$

$$5 \times 12 = 60 \quad 60 \div 12 = 5$$

OK for numbers less than 100 ! But what about *Rough Work* lists and making *estimates* when doing **long division**?

DIVISION

on Calculators and SCRATCH



If you're not sure of your division or multiplication **number facts** you could use a calculator or a computer (if you had one handy and if you know how to use it)! Here is a picture of a basic calculator.

$$42 \div 6 = ?$$

Be particular in using the words '**by**' and '**into**' with division. 42 divided **by** 6 is not the same as 42 **into** 6 (which doesn't **go!**)

The forward slash is often used for **division** on a calculator.

$$42 / 6 = 7$$

DIVISION is repeated SUBTRACTION

Show repeated subtraction with a Calculator

Not everyone knows this trick.

It illustrates division as repeated subtraction.

$$42 \div 6 = ?$$

Press **42 subtract 6 equals**. Press equals 7 times. Each time you subtract 6.

$$42 - 6 = 36$$

$$36 - 6 = 30$$

$$30 - 6 = 24$$

$$24 - 6 = 18$$

$$18 - 6 = 12$$

$$12 - 6 = 6$$

$$6 - 6 = 0$$

$$42 \div 7 = ?$$

Press **42 subtract 7 equals**. Press equals 6 times. How many subtractions?

$$42 - 7 = 35$$

$$35 - 7 = 28$$

$$28 - 7 = 21$$

$$21 - 7 = 14$$

$$14 - 7 = 7$$

$$7 - 7 = 0$$

$$180 \div 15 = ?$$

Use subtracting to find **how many times 15 are in 180**.

$$180 - 15 = 165$$
 Count each press = until 0 How many subtractions?

DIVISION is reverse of MULTIPLICATION

What **key** was pressed?

$$9 * ? = 72$$

What **key** was pressed?

$$8 * ? = 40$$

What **keys** were pressed?

$$? * 13 = 195$$

What **key** was pressed?

$$128 * ? = 768$$

SCRATCH

Multiplication and Division reporters in the Operators palette

$$8 * 3 = 24$$

$$24 \div 3 = 8$$

What numbers should you input?

$$20 / ? = 4$$

$$45 / ? = 9$$

$$40 / ? = 8$$

$$? / 3 = 12$$

$$? / 8 = 6$$

$$? / 9 = 7$$

Long Division as Repeated Subtraction

$4 \overline{)20}$		$7 \overline{)56}$		$4 \overline{)63}$	<i>this is long division</i>
5		8		$15 \overline{)63}$	
as subtraction	20	as subtraction	56	- 60	subtraction
once	- 4	once	- 7	remainder	3
	16		49		once
twice	- 4	twice	- 7		- 15
	12		42		twice
three times	- 4	three times	- 7		- 15
	8		35		three times
four times	- 4	four times	- 7		- 15
	4		28		four times
five times	- 4	five times	- 7		↓
↓	0		21		4
5	nothing remains	six times	- 7		3
			14		$\frac{3}{63} = \frac{1}{21}$
		seven times	- 7		0.48 to 2 places
			7		
		eight times	- 7		
		↓	0		
		8	nothing remains		

Sometimes a part of the original dividend remains. There are different ways to deal with a remainder:

- leave it as a whole number
- make it a fraction of the original
- make it a decimal of the original

Long Division as division

Write a *rough work* list and use *estimation* when you are doing **Long Division**. In this calculation you are dividing by 26. An important number is **260** (10 times 26).

- (a) You can subtract 26 from **260** ten times. **194** is less than 260, so the answer be less than 10. Count up in multiples of 26 and make a list, until you pass number **194**.

$$\begin{array}{r} 7 \\ 26 \overline{)194} \\ -182 \\ \hline 12 \end{array}$$

The last number that's less than 194 is 182, the 7th multiple of 26.

The quotient is 7 and the a remainder is 12

Rough Work

$$\begin{array}{l} 26\checkmark \\ 52\checkmark \\ 78\checkmark \\ 104\checkmark \\ 130\checkmark \\ 156\checkmark \\ \rightarrow 182\checkmark \\ 208 \end{array}$$

- (b)

$$\begin{array}{r} 32 \\ 18 \overline{)587} \\ -54 \downarrow \\ \hline 47 \\ -36 \\ \hline 11 \end{array}$$

You can see that you do not need to go past 54 (the 3rd multiple of 18). Write 3 above the 8. Draw a little dash mark in the place above the 7 where the next number will be.

Subtract 54 from 58.

Take down the 7 (to give 47)

Your RW list tells you that 36 is the 2nd multiple of 18. Write 2 above 7.

Subtract 36 from 47.

The remainder is 11

Rough Work

$$\begin{array}{l} 18\checkmark \\ 36\checkmark \\ \rightarrow 54\checkmark \\ -72 \end{array}$$

Do these 2 calculations on the Division Machine.

Complete the following Division Calculations

(Check your findings in the Division Machine)

A. Short division

(a) $28 \div 4 = \boxed{?}$

(d) $54 \div 3 = \boxed{?}$

(g) $4 \overline{)168}$

(b) $48 \div 6 = \boxed{?}$

(e) $78 \div 6 = \boxed{?}$

$\boxed{?}$

(c) $81 \div 9 = \boxed{?}$

(f) $98 \div 7 = \boxed{?}$

(h) $3 \overline{)225}$

$\boxed{?}$

B. Long division: Write a rough work list and use estimation.

Leave any remainder as whole number.

(a) $12 \overline{)156}$ = $\boxed{?}$ rem.

RW

(b) $15 \overline{)260}$ = $\boxed{?}$ rem.

RW

(c) $23 \overline{)323}$ = $\boxed{?}$ rem.

RW

(d) $37 \overline{)554}$ = $\boxed{?}$ rem.

RW

(e) $320 \div 64 = \boxed{?}$ rem.

(f) $643 \div 28 = \boxed{?}$ rem.

Division Gets More Interesting (Use the Division Machine)

A. Learning from Remainders

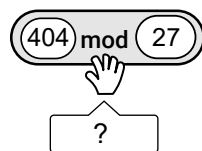
(1) $265 \div 44 = \boxed{?}$

- (a) What is the remainder in this division? *It's a number very close to the divisor.*
- (b) What is the **nearest** number to 265 that will have no remainder when divided by 44 ?
- (c) What is the next number **after** 265 that will have no remainder when divided by 44 ?
- (d) How many times will you be able to divide it by 44 ?

(2) $404 \div 27 = \boxed{?}$

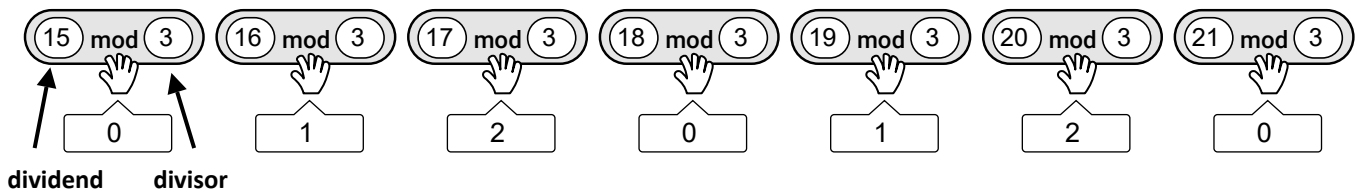
- (a) What is the remainder in this division? *It's a number very close to the divisor.*
- (b) What is the **nearest** number to 404 that will have no remainder when divided by 27 ?
- (c) What is the next number **after** 404 that will have no remainder when divided by 27 ?
- (d) How many times will you be able to divide it by 27 ?

(e) Here is an interesting SCRATCH Operator.



Fill in the numbers. Click **mod**.
What info is it telling you?

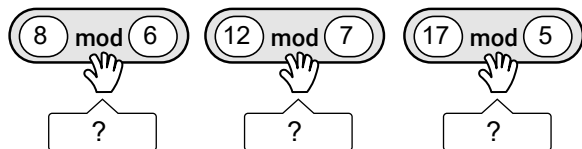
(3) Have you made the **mod** connection? If not, look at some easier division examples:



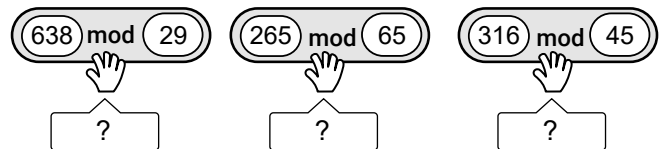
B. **Mod** is a special remainders block in SCRATCH. It is very useful in many situations.

Try these: Just fill the input windows and click. Try the Division Machine also.

Easy *mods* you'd do in your head!

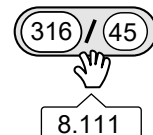


Not as easy. SCRATCH will tell you what the *mod* is.



What is the **nearest** number to each dividend that will divide evenly by the divisor?
Mod doesn't tell us what the quotient is? **Just use the division Operator.**

The remainder will be shown as a decimal.



C. **Mod** is very useful in many situations.

- (a) Every **EVEN** number reports a mod of **0** when divided by **2**
- (b) Every **ODD** number reports a mod of **1** when divided by **2**

